Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 2.

Due date: **Tuesday, 17.11.15.**

Programming Exercise 1. (Characteristics)

In the previous exercise sheet it was seen that the initial value problem

$$\begin{cases} u_t + uu_x &= 0, \quad t > 0,\ x \in \mathbb{R}, \\ u(x,0) &= u_0(x), \quad x \in \mathbb{R}. \end{cases} \tag{1}$$

has a unique solution if the implicit equation

$$x = x(y,t) := y + u_0(y)t, \tag{2}$$

in the unknown y has a unique solution $y(x,t)$ then

$$u(x,t) = u_0(y(x,t)). \tag{3}$$

a) Write a program `plot_char(y_l, y_r, t_i, t_e)` that given the initial condition $u_0(x)$, plots the characteristic lines (2) for $y \in$ `[y_l,y_r]` and $t \in$`[t_i,t_e]`.

b) Extend the program so it also sketches a solution (3) by dragging the initial data $u_0(x)$ along the characteristic lines (2). That is, plot the triples $(x(y,t), t, u_0(y))$.

Test your program with for following initial data functions

$$u_0(x) = \exp(-4(x-1)^2).$$

$$u_0(x) = \begin{cases} 1 & \text{if} \quad x \le 0, \\ 0 & \text{if} \quad x > 0. \end{cases}$$

$$u_0(x) = \begin{cases} 1 & \text{if} \quad x \le 0, \\ 1 - x & \text{if} \quad 0 < x < 1, \\ 0 & \text{if} \quad x \ge 1. \end{cases}$$

In all cases take $y \in [-1, 3]$ and $t \in [0, 2]$.

(6 points)

Programming Exercise 2. (2D Riemann Solver)

Write a program `Riemann_2D_solv(A,q_l,q_r)` that given any 2×2 matrix A and states q_l and q_r solves the Riemann problem and plots the solutions $q^1(x,t)$ and $q^2(x,t)$ as functions of x for some fixed time t. Test it out with

a) $A = \begin{pmatrix} 2 & 1 \\ 10^{-4} & 2 \end{pmatrix}$, $q_l = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $q_r = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

b) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $q_l = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $q_r = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

(4 points)