Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 8.

Due date: **Thursday, 7.1.16.**

Programming Exercise 1. (Flux limiters)

Write a program that implements the seven flux limiters that one can plug into the method

$$Q_i^{n+1} = Q_i^n - \nu \begin{Bmatrix} Q_i^n - Q_{i-1}^n \\ Q_{i+1}^n - Q_i^n \end{Bmatrix} - \frac{1}{2}\nu(1-\nu)\left(\phi\left(\theta_{i+\frac{1}{2}}^n\right)\Delta Q_{i+\frac{1}{2}}^n - \phi\left(\theta_{i-\frac{1}{2}}^n\right)\Delta Q_{i-\frac{1}{2}}^n\right) \tag{1}$$

(equation (2.2.30) in the lecture notes) to approximate the solution of the linear advection equation

$$\begin{cases} q_t + \bar{u} q_x = 0, & t > 0,\ x \in [0, 10], \\ q(x, 0) = q_0(x), & t = 0. \end{cases} \tag{2}$$

Use periodic boundary conditions and the following initial data,

$$q_0(x) = \begin{cases} \exp(-4(x-1)^2), & \text{if } x \in [0, 2], \\ 1, & \text{if } x \in [4, 6], \\ 0, & \text{else,} \end{cases} \qquad q_0(x) = \begin{cases} p(x), & \text{if } x \in [1, 5], \\ 0, & \text{else,} \end{cases} \tag{3}$$

where $p(x) = -\frac{1}{5}(x-3)^2(x-1)(x-2)(x-4)(x-5)$.

Requirements:

- The user should be able to choose the desired flux limiter and initial condition.
- The code should work for both cases: $\bar{u} > 0$ and $\bar{u} < 0$.
- All these methods are conservative. Add a mass conservation check after each solution update.
- The output of your program should be an animation showing the exact and the numerical solution evolving over time.

(10 points)