Numerical Algorithms

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Exercise Sheet 2.

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Exercise 1. (Gauss Theorem continued)

Let $\Omega \subset \mathbb{R}^d$ be an open, bounded domain with piecewise smooth boundary $\partial\Omega$. Let $v \in \mathcal{C}^1(\overline{\Omega}; \mathbb{R}^d)$ a vector field, $A \in \mathcal{C}^1(\overline{\Omega}; \mathbb{R}^{d\times d})$ a tensor field. Using a result from Exercise 1 of Sheet 1 show that

$$\int_\Omega A\colon \nabla v = -\int_\Omega v \cdot \nabla \cdot A + \int_{\partial\Omega} v \cdot A\nu$$

where ν is the outside normal vector, ∇v is the Jacobian matrix of v, ': ' denotes the Frobenius inner product and we define

$$B\colon C \coloneqq \sum_{i,j=1}^{d} b_{ij} c_{ij} \qquad \text{for } B, C \in \mathbb{R}^{d\times d},$$

$$(\nabla \cdot A)_i \coloneqq \sum_{j=1}^{d} \partial_j a_{ij} \qquad \text{for } i = 1, \dots, d.$$

(6 Points)

Exercise 2. (Higher Regularity in 1D)

Let $I = [a, b] \subset \mathbb{R}$, $f \in L_2(I)$, $u \in H_0^1(I)$ the weak solution of the Poisson equation

$$-u'' = f \quad \text{on } I$$

with zero Dirichlet boundary conditions. Show that $u \in H^2(I)$.

(6 Points)