# Numerical Algorithms

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## Exercise Sheet 12.

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**Exercise 1.** (Morton Order With Coordinates)

For $d, \ell, L \in \mathbb{N}_{>0}$, $\ell \leq L$ let $(\ell, I_L)$ with $I_L \in [0, 2^{dL}) \cap \mathbb{N}$ a valid mesh element in Morton representation and $(\ell, (x_j))$ with $x \in [0, 2^L)^d \cap \mathbb{N}^d$ a valid mesh element in coordinate representation. For $i = 0, \ldots, 2^d - 1$ prove the following properties

a) $\texttt{Morton\_child\_id}\,(\texttt{Morton\_child}(\ell, I_L, i)) = i$

b) $\texttt{coord\_parent}\,(\texttt{coord\_child}\,(\ell, (x_j), i)) = (\ell, (x_j))$

c) $\texttt{coord\_successor}\,(\ell, (x_j)) = \texttt{Morton\_coord}(\texttt{Morton\_coord}^{-1}\,(\ell, (x_j)) + (1 \ll (L - \ell)))$

d) Propose a function `coord_sibling` $(\ell, (x_j), i)$ using bit operations and not relying on `coord_parent` or `coord_child`.

(1+2+5+4 Points)

**Exercise 2.** (Partition Ranges)

For $N, P \in \mathbb{N}_{>0}$ let $Q \in [0, N) \cap \mathbb{N}$ and $O \in [0, N)^{P+1} \cap \mathbb{N}^{P+1}$ a process offset array, so $O_p \leq O_q$ if $p < q$, $O_0 = 0$ and $O_P = N$. Propose a function `find_smallest` $(O, Q)$ that returns the smallest process rank $q$ with $O_q \geq Q$.

(0 Points)