



Numerical Algorithms

Winter semester 2013/2014
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Exercise Sheet 4.

Due date: **Tuesday, 19 November.**

Exercise 7. (local integrability)

Given $\Omega \in \mathbb{R}^d$ open and bounded, show the equivalence

$$\int_K |f| \, dx < \infty \text{ for all compact } K \subset \Omega \quad \Leftrightarrow \quad \int_{\Omega} |f\varphi| \, dx < \infty \text{ for all } \varphi \in C_c^\infty(\Omega).$$

(4 points)

Exercise 8. (Sobolev norms)

Given $\alpha > 0$ and $B_1(0)$ the open unit ball, find all p that satisfy $u \in W^{1,p}(B_1(0))$ with $u(x) = |x|^{-\alpha}$.

(3 points)

Exercise 9. (heat ball)

Compute the radius of the heat ball

$$E(x, t; r) \cap \{(y, s) \in \mathbb{R}^{d+1} \mid s = t_0\} \subseteq \mathbb{R}^d$$

restricted to some fixed $t_0 < t$.

(2 points)