Numerical Algorithms

Winter semester 2013/2014
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Exercise Sheet 5.

Due date: **Thursday, 28 November**.

Exercise 10. (famous inequalities)

Prove the following estimates. Hint: Use the convexity of $x \mapsto e^x$ for a), a) for b), and b) for c).

a) Young inequality: Let $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then, for all $a, b \geq 0$,

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}\,.$$

b) Hölder inequality: All functions $f \in L^p(\mathbb{R}^d)$ and $g \in L^q(\mathbb{R}^d)$ with $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$ satisfy

$$fg \in L^1(\mathbb{R}^d) \quad \text{and} \quad \|fg\|_{L^1(\mathbb{R}^d)} \leq \|f\|_{L^p(\mathbb{R}^d)}\,\|g\|_{L^q(\mathbb{R}^d)}\,.$$

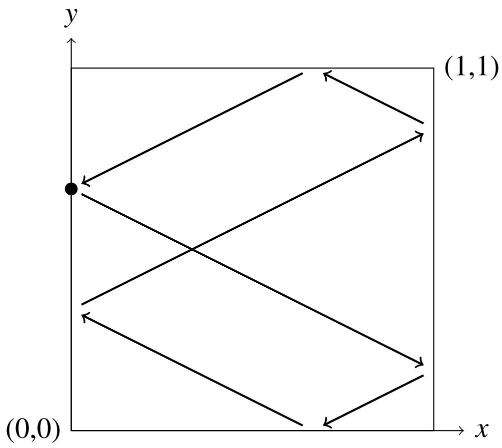
c) Minkowsky inequality: For any $p \in [1, \infty]$, the L^p-norm satisfies the triangle inequality

$$\|f + g\|_{L^p(\mathbb{R}^d)} \leq \|f\|_{L^p(\mathbb{R}^d)} + \|g\|_{L^p(\mathbb{R}^d)} \quad \text{for all } f, g \in L^p(\mathbb{R}^d).$$

(7 points)

Exercise 11. (time derivative)

Consider some totally boring Pong (constant speed, no friction, point-shaped ball) with solid boundaries everywhere and the ball's path depicted below. The start position is $(0, \frac{2}{3})$ and the next boundary contact is at $(1, \frac{1}{6})$. After 2 time units, the ball reaches the initial position again.



Show that the ball's location has a weak derivative with respect to time. Provided that the time and space units are seconds and meters respectively, compute the ball's speed.

(3 points)