Vorlesungsankündigung
Wintersemester 2014/15

Wavelet Frames

V5E2–Selected Topics in Science and Technology

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Content. Many real-world applications require the storage of continuous data on a discrete medium. Well-known examples are the commonly used jpeg-format for visual data or the mp3-format for audio data. Both formats are based on the decomposition of a continuous signal into a (possibly infinite) linear combination of translates and dilates of one single function ψ, the so-called wavelet. Ideally, the system $\{2^{j/2}\psi(2^j x - k)\}_{j,k\in\mathbb{Z}}$ is supposed to be an orthonormal basis in the Hilbert space of square integrable functions $L_2(\mathbb{R})$. The existence of such functions ψ is already ensured by the Haar function, which is well-localized, however less regular. The construction of general orthonormal wavelet bases with prescribed smoothness and localization properties is highly non-trivial. This lecture will present the necessary Fourier analytic techniques and focus on the most important representatives, e.g., spline wavelets, smooth Meyer wavelet, and C^k-smooth compactly supported Daubechies wavelets.

For practical issues it is often necessary to recover a signal from discrete samples of the continuous wavelet transform $\mathrm{CWT}f(a,b) := \langle f, \psi((\cdot - b)/a)\rangle$, $a > 0, b \in \mathbb{R}$. In other words, we aim at recovering a square integrable function f from a discrete set of wavelet coefficients, where we allow a certain redundancy in the wavelet system by sacrificing the orthonormality (and sometimes even the linear independence). We clarify under which conditions on the so-called wavelet frame this reconstruction is possible and stable. In addition, this lecture will provide relevant results from general frame theory in $L_2(\mathbb{R})$ as well as time-frequency analysis.

In the remaining time, we will study applications in approximation theory and function spaces. Moreover, we will comment on efficient decomposition and reconstruction algorithms and their use in signal processing. For interested listeners there will also be problem sheets available.

Requirements. Mathematical analysis, linear algebra, basic knowledge in functional analysis; helpful but not necessary is some background in Fourier analysis.

Date. Fridays, 10–12, SemR 6.020 (We6)

Literature

[1] A. Cohen. *Numerical analysis of wavelet methods*, volume 32 of *Studies in Mathematics and its Applications.* North-Holland Publishing Co., Amsterdam, 2003.

[2] O. Christensen. *Frames and bases – An introductory course*, volume 36 of the series *Applied and Numerical Harmonic Analysis*, Birkhäuser, Boston, 2008.

[3] I. Daubechies. *Ten lectures on wavelets*, volume 61 of *CBMS-NSF Regional Conference Series in Applied Mathematics.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992.

[4] K. Gröchenig. *Foundations of time-frequency analysis*, Birkhäuser, 2001.

[5] S. Mallat. *A wavelet tour of signal processing.* Elsevier/Academic Press, Amsterdam, third edition, 2009. The sparse way, With contributions from Gabriel Peyré.

[6] P. Wojtaszczyk. *A mathematical introduction to wavelets.* Cambridge University Press, 1997.