Scientific Computing II

Summer Semester 2014
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Excercise sheet 10.

Closing date **24.06.2014**.

Theoretical exercise 1. (Hierarchical error estimator: Condition Number [10 points])

For a sequence of linear Lagrange element spaces in two dimensions $\mathbb{V}_0 \subset \ldots \subset \mathbb{V}_L$ let the hierarchical decomposition be given by

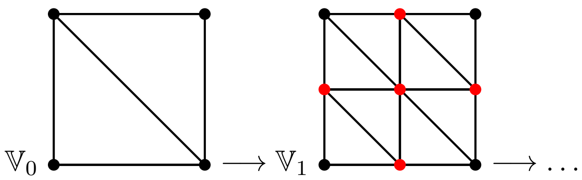
$$\mathbb{V}_l = \mathbb{V}_{l-1} \oplus \mathbb{W}_l$$

for all $l = 1, \ldots, L$, where $\mathbb{W}_l$ is the space of nodal basis functions for which the basis points appeared first on level l, i.e. $\mathbb{W}_l = \operatorname{span}\{\phi_i^{(l)}\}_{i \in B_l}$ with the notation from the lecture. Calculate the order of the condition number for the stiffness matrix

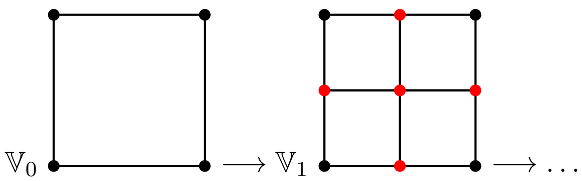
$$K_{11} = \left(a(\phi_i^{(l)}, \phi_j^{(l)})\right)_{i,j \in B_l}$$

of the standard Poisson problem with $a(u,v) = \int_{[0,1]^2} \nabla u \cdot \nabla v dx$ for uniform refinements of the following FE-spaces:

a) Linear triangle elements given by



b) Bilinear rectangular elements given by



Theoretical exercise 2. (Constant of the strenghtened Cauchy inequality [10 points])

For the examples in this exercise the meshes are the same as in Exercise 1. For $a(u,v) = \int_{[0,1]^2} \nabla u \cdot \nabla v dx$ and the hierarchical enrichment decomposition $\mathbb{V}_l := \mathbb{V}_{l-1} \oplus \mathbb{W}_l$ where $\mathbb{W}_l := \operatorname{span}\{\phi_i^{(l)} \mid i \in B_l\}$ is the complementary space of basis functions which are new on level l, calculate an as low as possible upper bound for the constant γ from the strenghtened Cauchy inequality for both linear triangular as well as bilinear quadratic elements.