Wissenschaftliches Rechnen II
(Scientific Computing II)

Sommersemester 2015
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Sheet 3 due date: **27. April 2015**

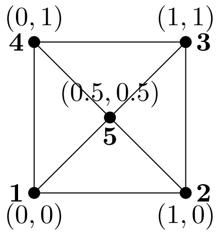
On 23. April: Preliminary session on FEM implementation (during the lecture)
On 27. April: Lab session in the computer lab 6.012

Download the software archive `P1FEM.zip` from the lecture website. The software can be used with Matlab or Octave.

Data structures. A regular triangulation is described by the structure array `T` containing

- `T.coords` = matrix with 2 columns whose rows describe the coordinates of the vertices
- `T.elems` = matrix with 3 columns whose rows describe the triangles (counterclockwise)
- `T.dirichlet` = matrix with 2 columns. The rows contain the endpoints of the edges of the Dirichlet boundary
- `T.neumann` = matrix with 2 columns. The rows contain the endpoints of the edges of the Neumann boundary

Example. *(square with pure Dirichlet boundary)*



$$\texttt{T.coords} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}, \texttt{T.elems} = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \end{pmatrix}, \texttt{T.dirichlet} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 1 \end{pmatrix}, \texttt{T.neumann} = []$$

Files.

- `refine.m` = `T=refine(T)` creates a uniform refinement of the triangulation `T`
- `P1FEM.m` = the P_1 finite element method for the Poisson equation in 2D with Dirichlet boundary
- `exampleSquare.m` = example on the unit square

Exercise 10. *(L-shaped domain)*

(a) Write the data structure `T` for a triangulation of the L-shaped domain $\Omega := (-1, 1)^2 \setminus ([0, 1] \times [-1, 0])$.

(b) Plot the convergence history for $-\Delta u = 1$ on the L-shaped domain (cf. Exercise 8; the exact solution satisfies $\|\nabla u\|^2 = 0.2140750232$). Compare the convergence rate with the output of `exampleSquare.m` for the unit square.

Exercise 11. *(convection-diffusion equation)*

(a) Implement the P_1 FEM for the convection-diffusion equation $-\varepsilon\Delta u + \beta \cdot \nabla u = f$ (extend the existing program `P1FEM.m`).

(b) Consider the unit square $\Omega = (0,1)^2$ with homogeneous Dirichlet boundary conditions and the right-hand side f according to the exact solution

$$u(x) = \left(\frac{e^{r_1(x_1-1)} - e^{r_2(x_1-1)}}{e^{-r_1} - e^{-r_2}} + x_1 - 1 \right) \sin(\pi x_2)$$

with

$$r_1 = \frac{-1 + \sqrt{1 + 4\varepsilon^2\pi^2}}{-2\varepsilon} \quad \text{and} \quad r_2 = \frac{-1 - \sqrt{1 + 4\varepsilon^2\pi^2}}{-2\varepsilon}.$$

Run numerical computations for the following parameters

(i) $\varepsilon = 0.1$ and $\beta = (1,0)^T$.

(ii) $\varepsilon = 0.001$ and $\beta = (1,0)^T$.