Scientific Computing II

Sommersemester 2019
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Exercise Sheet 12.

Due date: **- No deadline -**.

This is a recap exercise sheet that you could use to check your understanding. The questions are brief and direct; you must be ready to answer them quickly and clearly. Every abuse of notation / ambiguity is done on purpose: you have to use your knowledge to clarify what I meant. A statement concerning an object, e.g. "0) Bilinear form a;" means that you have to state its definition and main properties. The shortcut "VS" means "compare".

1 Weak formulation, spaces, some functional analysis

1) What is the structure of the PDE operator L required for our Finite Element setting?
2) What would you do if you encountered non-homogeneous boundary conditions?
3) Lax-Milgram theorem VS Weak formulation;
4) Lax-Milgram theorem VS inf-sup conditions;
5) What is a Galerkin method?
6) What is the energy scalar product/norm? If it is equivalent to the Hilbert product, why still use it?
7) Given the weak formulation, derive the stability $\|u_h\|_m \leq \alpha^{-1}\|f\|_{-m}$
8) Cea's lemma VS Generalized Cea's lemma;
9) Stiffness matrix: definition and main properties;
10) Show a relevant and useful example of Gelfand triple;
11) Ω bounded. Are H^m, H_0^m always Hilbert?
12) I say: U, V Hilbert. $L : U \to V'$, $a : U \times V \to \mathbb{R}$. Relate the two.
13) Suppose to have an iso $L : U \to V'$ is the setting that you know. And now?
14) Give a valid reason to study saddle point problems;
15) Idea and usefulness of Brezzi's splitting theorem;

2 Iterative methods

16) FE setting. Why linear algebra? Why iterative instead of direct?

17) What's the (usual) underlying principle beyond a (generic) iterative method?

18) Intuitively, why $\rho(G) < 1$ relates to the convergence of G^k?

19) Key hypothesis for Jacobi, Gauss-Seidel;

20) Main limitation of SOR. Why does it not represent a problem in the FE setting?

21) Show a relevant example where SOR is sensitively faster that Jacobi/Gauss-Seidel;

22) What is the underlying idea for multigrid methods?

23) Sketch the algorithm for 2-grid;

24) Restriction and prolongation operators. What are they? Relation?

25) Explain A^s. Key property used for multigrid convergence?

26) What are the two main steps involved for checking multigrid convergence?

27) Algebraic multigrid VS Geometric (i.e. ordinary) multigrid;

3 Finite Elements

28) Precise definition of Finite Element, connection with the weak formulation;

29) Give 3 examples of Finite Elements;

30) Let Ω be our domain. But suppose it *not* to be a FE. What would you do? How would you ensure to obtain a space regular enough for approximating, e.g. $H^1(\Omega)$?

31) Why polynomials? Give three good reasons;

32) How would you compute a global Stiffness matrix for a general domain? (not a FE)

33) What is a nodal basis? Suppose you use them to solve your PDE. So you solve a linear system, and obtain numbers. What do they mean? What if you used another basis?