Numerical Algorithms

Winter semester 2013/2014
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Exercise Sheet 6.   Due date: **Thursday, 19 December**.

Programming Exercise 1. (FE solver)

Solve the Laplace equation in 2D as follows. Given $\Omega = (0,1)^2 \subset \mathbb{R}^2$ and $f \in L^2(\Omega)$, approximate $u \in H_0^1(\Omega)$ such that

$$\int_\Omega \nabla u \nabla v \, \mathrm{d}x = \int_\Omega f v \, \mathrm{d}x \quad \text{for all } v \in H_0^1(\Omega).$$

In order to do this, choose a discretization of the unit square, for example by piecewise linear triangles or bilinear squares. Assemble the stiffness and mass matrices in the compressed sparse storage format provided by the Linear-Algebraic System Interface (IJ) of the Hypre software library (linked on the website). Installation and use of the interface are explained in the user manual.

Implement the conjugate gradient method using the matrix-vector-product from the Hypre library, which operates on the sparse storage format.

Design an example problem with a known analytic solution. Assess the discrete solution for different mesh sizes by computing the L^2- and H^1-error and counting the iterations of your solver.

Addendum

Compute the L^2- and H^1-error using the mass and stiffness matrices on the one hand, and on the other hand using a higher order quadrature. Proceed as follows.

0. Choose a quadrature of at least third order, and extend it by a tensor product (Fubini) onto the 2D square. If you are using triangles, transform the quadrature onto a triangle by contracting one edge to a point and applying the transformation rule.

1. Implement a loop over all elements.

2. On each element, iterate over the quadrature points.

3. At each quadrature point, evaluate the Finite Element function of the approximation and substract the evaluated exact solution. The increase of the squared integral is then the product of squared difference and quadrature weight.

For testing, integrate the constant function $u \equiv 1$ and assess the integral value. Compare the observed convergence rates of the two methods.

(10 points)

Submit your solutions via email to morgenstern@ins.uni-bonn.de .
Deadline is the tutorial on Thursday, 19 December.