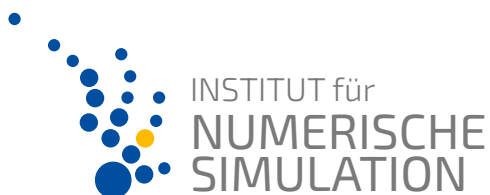


9th Chinese-German Workshop on Computational and Applied Mathematics

5 – 9 October 2026

Mathematikzentrum
Endenicher Allee 60
53115 Bonn

Organized by
University of Bonn
Institute for Numerical Simulation



Workshop Venue

Directions: Mathematikzentrum Bonn

Endenicher Allee 60, 53115 Bonn
Lipschitz-Saal, Room 1.016

From: Bonn Hauptbahnhof (main train station)

On foot

Approx. 20 min (1 km). Leave Bonn Hauptbahnhof via the rear exit (south-west) onto Quantiusstraße. Turn right and follow the road — it becomes Colmantstraße, then continues straight into Endenicher Allee. After about 1 km you'll see the Mathematikzentrum on the right.

By bus

Take bus line 604, 605, 606, 607 towards Endenich/Duisdorf and get off at **Kaufmannstraße**, right in front of the building (approx. 5 min).

Note: the lower/ground-level entrance leads into the library. To reach the Lipschitz-Saal directly, use the outside staircase on the Kaufmannstraße side — it takes you straight up to the first floor, right next to Room 1.016.

Lunch Options

University canteen

- **Mensa CAMPO** (Campusmensa Poppelsdorf), Endenicher Allee 19 – directly across the street. Mon–Fri 11:30–14:30. Salad bar, homemade pizza, daily changing dishes. *Cashless payment only* (Mensa card, debit or credit card).
- **Café CAMPO**, ground floor of the Mensa building – coffee, sandwiches, savoury and sweet snacks, outdoor terrace. Mon–Fri 08:00–15:00.

Clemens-August-Straße (approx. 10 min walk)

- **Buena Vida Havanna** (No. 1) – restaurant/bar/café; pasta, pizza, salads. Daily from 11:00.
- **El Tarascon** (No. 2–4) – Argentinian steakhouse; lunch Tue–Fri 12:00–14:30.
- **Esskalation** (No. 7a) – vegan café/restaurant; daily from 11:00.
- **Hans im Glück** (No. 9) – burgers, incl. vegetarian/vegan; daily from 12:00.
- **Nennillo** (No. 19) – Neapolitan pizza, lunch menu; daily from 12:00.
- **Schloss-Café** (No. 21) – café and pastry shop; Tue–Sun from 8:00 (closed Mon).
- **Tibet Imbiss** (No. 39) – Tibetan noodles and dumplings, quick and inexpensive; Tue–Sun from 11:00 (closed Mon), cash only.
- **Voigt** (No. 48) – bakery/café; daily from early morning.
- **Pastarei** (No. 53) – fresh Italian pasta; Mon–Sat from 12:00.
- **Kurt der Kaffeeröster** (No. 55) – specialty coffee, cakes, small dishes; daily 9:00–18:00.
- **Sapore** (No. 67) – Italian; Tue–Sun from 12:00 (closed Mon).
- **COBAMI** (No. 69) – Vietnamese/Japanese, pho and sushi; daily from 11:30.

Workshop Dinner

Directions: Gasthaus Im Stiefel

Bonngasse 30, 53111 Bonn

From: Mathematikzentrum Bonn, Endenicher Allee 60, 53115 Bonn

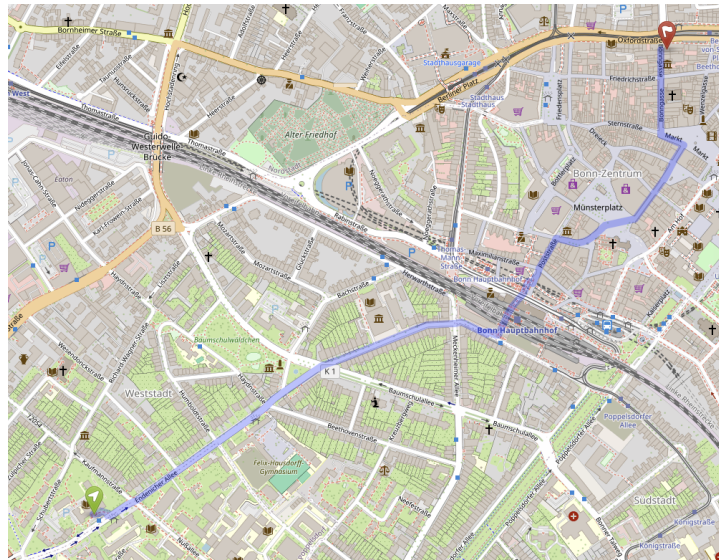
Distance: approx. 2.3 km (approx. 30 min walk)

On foot

From the Mathematikzentrum, follow Endenicher Allee towards the city center / Bonn Hauptbahnhof. Continue into the pedestrian zone; Gasthaus Im Stiefel is located on Bonngasse, right next to the Beethoven House.

By bus

Bus stop Kaufmannstraße is directly in front of the Mathematikzentrum. Take lines 604, 605, 606 or 607 to Bonn Hauptbahnhof, then walk about 10 minutes through the city center to Bonngasse.



Acknowledgements

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Funded by the TRA Modelling (University of Bonn) as part of the Excellence Strategy of the federal and state governments.

Program

Monday, Oct 5, 2026

08:45 – 09:00	Opening by Joscha Gedicke
09:00 – 09:30	Chair: Joscha Gedicke Jun Hu : Discretizing linearized Einstein–Bianchi system by symmetric and traceless tensors
09:30 – 10:00	Carsten Carstensen : Nonconforming approximation of Ginzburg-Landau-type semilinearities
10:00 – 10:30	Coffee break
10:30 – 11:00	Chair: Rui Ma Johannes Storn : The singular Zienkiewicz tetrahedron
11:00 – 11:30	Joscha Gedicke : A generalized hessian-based error estimator for the 2d biharmonic problem via SIPDG
11:30 – 13:30	Lunch break
13:30 – 14:00	Chair: Carsten Carstensen Rui Ma : Adaptive $H(\text{divdiv})$ mixed finite element methods for the Kirchhoff-Love plate bending problem
14:00 – 14:30	Yizhou Liang : Finite Element Complexes with Traces Structures: A unified framework for cohomology and bounded interpolation
14:30 – 15:00	Coffee break
15:00 – 15:30	Chair: Dongjie Liu Barbara Zwicknagl : Variational models for pattern formation in materials: From frustrated spin systems to shape-memory alloys
15:30 – 16:00	Christian Rieger : Semi-Lagrange methods on manifolds

Tuesday, Oct 6, 2026

09:00 – 09:30	Chair: Christoph Lehrenfeld Chunmei Su : Temporal high-order parametric finite element methods for geometric flows
09:30 – 10:00	Buyang Li : Finite element methods for isometric embedding of Riemannian manifolds
10:00 – 10:30	Coffee break
10:30 – 11:00	Chair: Buyang Li Christoph Lehrenfeld : Discrete streamfunction and harmonic fields for incompressible flows on general surfaces
11:00 – 11:30	Oliver Sander : Numerical Analysis of Geometric FE Discretizations of Cosserat Materials
11:30 – 13:30	Lunch break
13:30 – 14:00	Chair: Fleurianne Bertrand Dongjie Liu : The nonconforming finite element methods for equations with p-Laplace structure
14:00 – 14:30	Xuejun Xu : Interior Eigensolvers with Domain Decomposition
14:30 – 15:00	Coffee break
15:00 – 15:30	Chair: Xuejun Xu Daniel Peterseim : Metric-driven numerical methods for nonlinear eigenvalue problems
15:30 – 16:00	Fleurianne Bertrand : Local Quasi-equilibration of fluxes with the Least-Squares method

16:00 – 16:30	Axel Voigt: Arrested coarsening in two-phase systems and characterization of resulting morphologies
16:30 – 17:00	Peipei Lu: Two-level hybrid Schwarz methods for Helmholtz equation with high wavenumber

Wednesday, Oct 7, 2026

09:00 – 09:30	Chair: Guozhi Dong
09:30 – 10:00	Malte Braack: Mapped coercive real- and complex-valued PDEs
10:00 – 10:30	Lars Diening: Regularity of discrete solutions
10:30 – 11:00	Chair: Lars Diening
11:00 – 11:30	Junming Duan: An Active Flux Method for Compressible Navier-Stokes Equations
11:30 – 13:30	Chupeng Ma: A unified framework for low-rank approximations to multiscale PDEs with optimal error bounds
13:30 – 16:00	Time for free discussions

Thursday, Oct 8, 2026

09:00 – 09:30	Chair: Xiaoping Xie
09:30 – 10:00	Moritz Hauck: Numerical analysis of dynamic spatial network models
10:00 – 10:30	Robert Scheichl: Localised Model Reduction via Generalised Finite Elements
10:30 – 11:00	Chair: Peipei Lu
11:00 – 11:30	Thomas Wick: Parallel Monolithic Matrix-Free Solvers for Coupled Problems
11:30 – 13:30	Guido Kanschat: Efficient GPU implementations of multigrid methods
13:30 – 14:00	Chair: Guido Kanschat
14:00 – 14:30	Xiaoping Xie: A Reynolds-semi-robust globally divergence-free E-HDG method for variational data assimilation problems of Navier-Stokes Equations
14:30 – 15:00	Hongpeng Sun: A stochastic preconditioned Douglas-Rachford splitting method for saddle-point problems
15:00 – 15:30	Chair: Chunmei Su
15:30 – 16:00	Patrick Henning: Finite Element Resolution of Nonlinear Ground-State Problems with Small Scales
16:00 – 16:30	Ngoc Tien Tran: Finite element discretizations for fully nonlinear Hamilton-Jacobi-Bellman equations
16:30 – 17:00	Christiane Helzel: Fully discrete Active Flux methods for the Euler equations
17:00 – 19:00	Thomas Richter: Hybrid Neural Network / Finite Element Approaches for Solving Partial Differential Equations
19:00 – 21:00	Workshop Dinner

Friday, Oct 9, 2026

09:00 – 09:30	Chair: Thomas Wick
09:30 – 10:00	Guozhi Dong: Second-order dynamics: An old bottle of new wine
10:00 – 10:30	Ye Zhang: Neural network yields regularization for ill-posed inverse problems

10:00 – 10:30	Coffee break
10:30 – 11:00	Chair: Wei Gong Ira Neitzel: On some aspects of four-dimensional variational data assimilation problems in low-regularity spaces
11:00 – 11:30	Michael Hinze: Provably convergent PDE constrained shape optimization
11:30 – 13:30	Lunch break
13:30 – 14:00	Chair: Malte Braack Wei Gong: A penalty approach for topology optimization problems
14:00 – 14:30	Roland Herzog: Preconditioned solution of a magnetostatic optimal control problem
14:30 – 15:00	Closing & Coffee break

Participants

- Bertrand, Fleurianne (TU Chemnitz)
- Braack, Malte (Christian-Albrechts-Universität zu Kiel)
- Carstensen, Carsten (Humboldt-Universität zu Berlin)
- Diening, Lars (U Bielefeld)
- Dong, Guozhi (Central South University)
- Duan, Junming (The Chinese University of Hong Kong (Shenzhen))
- Gedicke, Joscha (Universität Bonn)
- Gong, Wei (Chinese Academy of Sciences)
- Hauck, Moritz (Karlsruher Institut für Technologie (KIT))
- Helzel, Christiane (HHU Düsseldorf)
- Henning, Patrick (RU Bochum)
- Herzog, Roland (Universität Heidelberg)
- Hinze, Michael (Universität Koblenz)
- Hu, Jun (Peking University)
- Kanschat, Guido (Universität Heidelberg)
- Lehrenfeld, Christoph (Universität Göttingen)
- Li, Buyang (The Hong Kong Polytechnic University)
- Liang, Yizhou (Jilin University & University of Oxford)
- Liu, Dongjie (Shanghai University)
- Lu, Peipei (Soochow University)
- Ma, Chupeng (Great Bay University)
- Ma, Rui (Beijing Institute of Technology)
- Neitzel, Ira (Universität Bonn)
- Peterseim, Daniel (Universität Augsburg)
- Richter, Thomas (Universität Magdeburg)
- Rieger, Christian (Universität Marburg)
- Sander, Oliver (TU Dresden)
- Scheichl, Robert (Universität Heidelberg)
- Storn, Johannes (Universität Leipzig)
- Su, Chunmei (Tsinghua University)
- Sun, Hongpeng (Renmin University of China)
- Tran, Ngoc Tien (Universität Augsburg)
- Voigt, Axel (TU Dresden)

- Wick, Thomas (Universität Hannover)
- Xie, Xiaoping (Sichuan University)
- Xu, Xuejun (Tongji University)
- Zhang, Ye (Shenzhen MSU-BIT University)
- Zwicknagl, Barbara (Humboldt-Universität zu Berlin)

Abstracts

Fleurianne Bertrand

Local Quasi-equilibration of fluxes with the Least-Squares method

Equilibrated flux and stress reconstructions play a central role in guaranteed a posteriori error estimation, but their classical construction relies on compatible mixed finite element spaces and associated inf-sup conditions. In many practical situations, however, such compatibility is unavailable or undesirable, for instance when using primal discretizations or non-mixed approximation spaces.

In this work, we propose a quasi-equilibrated reconstruction strategy that yields fluxes which satisfy equilibrium conditions up to higher-order terms, without requiring a mixed formulation. Starting from a given discrete primal solution, the flux is reconstructed by solving local patch-wise least-squares problems that simultaneously control the deviation from the discrete gradient and the equilibrium residual. The local reconstructions are combined by a partition-of-unity operator, resulting in a globally defined $H(\text{div})$ -conforming flux.

Although exact local equilibrium is not enforced, we show that the remaining equilibrium defect is asymptotically negligible and does not affect reliability of the resulting a posteriori error estimator. In particular, the estimator is reliable with respect to the natural least-squares norm and asymptotically sharp for the energy error. The proposed approach avoids mixed compatibility requirements and inf-sup conditions, while retaining the essential features of equilibrated estimators for adaptive refinement.

Malte Braack

Mapped coercive real- and complex-valued PDEs

We extend the recently introduced theory of mapped coercive operators from real to complex Banach spaces. While classical surjectivity results for nonlinear operators, such as the Browder-Minty theorem, rely on monotonicity in addition to coercivity, mapped coercivity provides a more general framework that does not require monotonicity. We formulate the appropriate notion of mapped coercivity in complex Banach spaces by employing the real part of the duality pairing and establish a corresponding surjectivity theorem for weakly continuous operators on reflexive complex Banach spaces. The resulting theory generalizes the existing framework for real Banach spaces and complements the classical existence theory for monotone operators in the complex setting. As applications, we prove existence of weak solutions for several nonlinear complex-valued partial differential equations, including the stationary nonlinear Schrödinger equation, the stationary complex Ginzburg-Landau equation, and the complex Maxwell problem with solution dependent coefficients. These examples demonstrate that mapped coercivity yields a unified existence theory for nonlinear operator equations in complex Banach spaces beyond the classical monotone framework.

Carsten Carstensen
(joint with Lara Théallier)

Nonconforming approximation of Ginzburg-Landau-type semilinearities

The Landau-de Gennes model for nematic liquid crystals provides computational challenges

within a non-convex minimization problem. The associated Euler-Lagrange equations form a semilinear second-order elliptic boundary value problem with reduced regularity in non-convex domains as well as additional topological singularities called vortices. The energy landscape in this non-convex minimisation problem is unexpectedly rich with many stationary points of the energy functional and severe difficulties for local solve. Nonconforming Crouzeix-Raviart finite elements allow for lower energy bounds in the asymptotic range of sufficiently fine meshes and immediate approximation of inhomogeneous Dirichlet data.

The presentation departs with 2D computational benchmark examples and explains the origin of vortex singularities for Dirichlet data of non-zero winding number for larger Ginzburg parameters ℓ for this is a novel aspect in the mandatory adaptive mesh-refining. A priori existence of discrete solutions and their weak or strong (global) convergence towards stationary points along subsequences is illuminated. An asymptotic a priori and a posteriori local error analysis with optimal rates for appropriate adaptive algorithms concludes the presentation.

- [1] Carstensen C., Dond A. K., Maity R. R. and Nataraj N. *Crouzeix-Raviart finite elements for lower energy bounds in Ginzburg-Landau-type minimisation*, Comput. Math. Appl. 214 (2026), 27–50.
- [2] Carstensen C., Dond A. K., Maity R. R., Nataraj N. and Théallier, L. *Adaptive Crouzeix-Raviart finite elements for a non-convex Ginzburg-Landau model for nematic liquid crystals*, CMAME 52 (2026), 118646.
- [3] Carstensen C., Dond A. K., Maity R. R., and Théallier L. *Adaptive Crouzeix-Raviart finite elements in Ginzburg-Landau-type minimisation*, work in progress (2026).

Lars Diening

Regularity of discrete solutions

We study the regularity of finite element solutions in the context of elliptic equations. We consider linear, non-linear and weighted equations. We look at Hölder continuity as well as higher Sobolev regularity.

Guozhi Dong

Second-order dynamics: An old bottle of new wine

Second-order dynamics serve as mathematical models in mechanics to describe dispersion and dissipation of energy in a class of inertial systems. We are trying to figure out some of the new perspective of second-order dynamics. Our investigation indicates that it can also be a powerful tool to develop numerical methods for solving variational and inverse problems in Hilbert spaces, which provides novel connections between numerical methods of dissipative hyperbolic equations and optimization in infinite dimensional spaces. This talk will show some of our recent development in this aspect.

Junming Duan

An Active Flux Method for Compressible Navier-Stokes Equations

The active flux (AF) method is a compact high-order finite volume method, where cell averages and point values at cell interfaces are evolved simultaneously. This paper develops a high-order AF method for compressible Navier-Stokes equations. The evolution of cell averages follows the standard finite volume method, ensuring conservation, while the point values are updated using finite difference operators. To maintain the original compact stencil of the AF methods, the proposed method is built on compact finite difference operators for second derivatives. Several numerical experiments validate the theoretical results and efficacy of the method.

Joscha Gedicke

A generalized hessian-based error estimator for the 2d biharmonic problem via SIPDG

This talk presents a new error estimation framework for the symmetric interior penalty discontinuous Galerkin (SIPDG) discretization of the two-dimensional biharmonic problem. The central contribution is a novel decomposition of an error measure based on a generalized Hessian into two distinct components: one measuring the conformity and the other the nonconformity of the scheme. Leveraging this splitting, we design a new error estimator that is provably reliable and efficient for polynomial degrees $p \geq 3$, notably without the need for any DG stabilization terms. Furthermore, we show that this estimator can be bounded from above by the standard DG residual-based error estimator. Numerical experiments are provided to validate the theoretical predictions, demonstrating the efficiency of the proposed estimator for all polynomial degrees $p \geq 2$.

Wei Gong

A penalty approach for topology optimization problems

We propose a novel penalty method framework for compliant mechanism problems, incorporating a convex nonlocal perimeter approximation scheme. We rigorously analyze the existence of solutions to the optimization problem derived from the penalty method. Furthermore, we establish that the discrete problem Gamma-converges to the continuous problem, ensuring consistency across scales. To solve the discrete problem, we develop a projected gradient method that guarantees strict monotonic descent of the objective function. Numerical experiments on the compliant mechanism and heat dissipation problems validate the effectiveness of the proposed method, with results supported by convergence analysis. This is a joint work with Yuanda Ye (AMSS, CAS).

Moritz Hauck

Numerical analysis of dynamic spatial network models

In this talk, we consider the numerical solution to dynamic beam network models that describe the elastic deformation of fiber-based materials. These models consist of elastodynamic equations for each network edge, coupled at nodes via suitable transmission conditions. Leveraging

the network structure, we present and analyze a hybridizable discontinuous Galerkin method with a number of globally coupled degrees of freedom, independent of the local polynomial degree and spatial resolution. To efficiently solve the resulting global systems at each time step, which are typically ill-conditioned, we introduce a two-level, overlapping, additive Schwarz preconditioner. Assuming certain conditions about the network, we analyze this preconditioner and demonstrate the uniform convergence of the resulting preconditioned conjugate gradient method.

Christiane Helzel

Fully discrete Active Flux methods for the Euler equations

The Active Flux method, originally proposed by P.L. Roe, is a relatively new finite volume method for hyperbolic conservation laws that uses cell averages and point values as degrees of freedom. Several versions of this method have recently been developed. We will present recent results of our group on the construction of third-order-accurate, fully discrete Active Flux methods with compact stencil in space and time. A crucial component of these methods is the evolution operator, which is used to update of the point values. We will discuss several multi-dimensional operators and present numerical results for the two-dimensional Euler equations.

This is joined work with Erik Chudzik and Amelie Porfetye (Düsseldorf) and Maria Lukacova and Zhuyan Tang (Mainz).

Patrick Henning

Finite Element Resolution of Nonlinear Ground-State Problems with Small Scales

Many nonlinear variational problems involve small spatial scales that have to be resolved accurately by a finite element discretization. A natural expectation is that the mesh size should be comparable to the smallest relevant physical length scale. We show that this intuition can fail when the underlying energy landscape becomes locally flat near a minimizer. We study this phenomenon for a class of nonlinear ground-state problems arising in the rapid-rotation regime of the Gross–Pitaevskii equation. The small parameter ε determines the size of vortex cores, suggesting the natural resolution condition $h \lesssim \varepsilon$. However, as the system approaches a critical regime, the stability of the ground state deteriorates and an additional, significantly stronger mesh constraint emerges. Our analysis relates the finite element approximation error to the local spectral stability of the continuous minimizer. In particular, we identify the first spectral gap of the Riemannian Hessian as a key quantity governing the required mesh resolution and prove conditions under which discrete minimizers are quasi-best approximations of exact ground states. We also derive the asymptotic H^1 -error estimate and discuss how the resulting theory explains the mesh resolution required to accurately capture vortex structures.

Roland Herzog

Preconditioned solution of a magnetostatic optimal control problem

We consider the preconditioned iterative solution for the optimal control of magnetostatic equations. The proposed method considers a proper regularization of the saddle-point forward system, featuring a positive-definite structure and preserving the divergence-free condition for the

state. On the basis of the resulting optimality system, we develop a specific preconditioner with upper and lower bounds for the eigenvalues with mild dependence on the regularization parameter and the mesh size. This property guarantees the desired robustness of the proposed preconditioner in the limit case, which is as well confirmed by our numerical tests.

Michael Hinze

**Provably convergent PDE constrained shape optimization
(joint with Klaus Deckelnick und Philip Herbert)**

We present a general shape optimisation framework based on the method of mappings in the Lipschitz topology. We propose and numerically analyse steepest descent (and Newton-like) minimisation algorithms for the numerical solution of the respective shape optimisation problems. In particular, we prove global convergence of steepest descent with Armijo stepsize rule in function space. To illustrate our approach we present a selection of PDE constrained shape optimisation problems and compare our findings to results from so far classical Hilbert space methods and recent p-approximations. If time permits, we also present a hybrid approach which combines a phasefield approach with our sharp framework.

Jun Hu

Discretizing linearized Einstein–Bianchi system by symmetric and traceless tensors

The Einstein-Bianchi system uses symmetric and traceless tensors to reformulate Einstein's original field equations. However, preserving these algebraic constraints simultaneously remains a challenge for numerical methods. This talk proposes a new formulation that treats the linearized Einstein-Bianchi system (near the trivial Minkowski metric) as the Hodge wave equation associated with the conformal Hessian complex. To discretize this equation, a conforming finite element conformal Hessian complex that preserves symmetry and tracelessness simultaneously is constructed on general three-dimensional tetrahedral grids, and its exactness is proven.

Guido Kanschat

Efficient GPU implementations of multigrid methods

Multigrid methods combine optimal algorithmic complexity with a hierarchy of computational kernels exhibiting rather different performance characteristics. On modern GPUs, their efficient realization therefore requires more than simply transferring a conventional CPU implementation to the accelerator.

In this talk, we discuss implementation principles for matrix-free geometric multigrid methods for high-order finite element discretizations. Particular attention is paid to vertex-patch smoothers, which provide robust convergence for high-order problems but pose challenging questions of data locality and parallelism. Exploiting tensor-product structure and fast diagonalization, the local patch problems can be solved at a cost comparable to matrix-free operator evaluation. At the same time, reorganizing the computation such that intermediate data remain in on-chip memory substantially reduces traffic to global memory. This leads to a performance regime in which not only device memory bandwidth, but also shared-memory bandwidth and access patterns become limiting factors.

Christoph Lehrenfeld

(joint work with Tim Brueers, Tim van Beeck and Max Wardetzky)

Discrete streamfunction and harmonic fields for incompressible flows on general surfaces

Incompressible flows on curved surfaces must be both divergence-free and tangential to the surface – two structural constraints that are hard to satisfy exactly at the discrete level. In the classical velocity-pressure setting the pressure enforces the divergence constraint as a Lagrange multiplier, giving a saddle point problem, while tangentiality is typically imposed only weakly, by penalties or a further multiplier.

We take the Helmholtz-Hodge decomposition as the guiding principle instead: on a general, possibly non-simply-connected surface, every tangential divergence-free field is the surface curl of a scalar streamfunction plus a harmonic field, the harmonic space having dimension equal to the first Betti number. Both constraints are thereby encoded in the ansatz rather than imposed on it, which removes the pressure, the saddle point structure and any penalty mechanism.

We follow two routes: a streamfunction-vorticity formulation of the surface Navier-Stokes and Euler equations, shown to be equivalent to the velocity-pressure formulation and hence well-posed [1]; and a fully discrete decomposition of the divergence-free $H(\text{div})$ -conforming BDM subspace into rotated gradients and discrete harmonic fields, yielding exactly tangential, point-wise divergence-free and pressure-robust high-order methods, with numerical results on a trefoil knot and a multiply connected sculpture surface [2].

[1] Brueers, Lehrenfeld, Wardetzky. arXiv:2512.20763 (2025).

[2] Brueers, Lehrenfeld, van Beeck, Wardetzky. Comput. Methods Appl. Mech. Engrg. 461 (2026), doi:10.1016/j.cma.2026.119201.

Buyang Li

Finite element methods for isometric embedding of Riemannian manifolds

The isometric embedding problem for Riemannian manifolds, which connects intrinsic and extrinsic geometry, is a central question in differential geometry with deep theoretical significance and wide-ranging applications. Despite extensive analytical progress, the nonlinear and degenerate nature of this problem has hindered the development of rigorous numerical analysis in this area. As the first step toward addressing this gap, we study the numerical approximation of Weyl's problem, i.e., the isometric embedding of two-dimensional Riemannian manifolds with positive Gaussian curvature into $\mathbb{R}^3$, by establishing a new weak formulation that naturally leads to a numerical scheme well suited for high-order finite element discretization, and conducting a systematic analysis to prove the well-posedness of this weak formulation, the existence and uniqueness of its numerical solution, as well as its convergence with error estimates. This provides a foundational framework for computing isometric embeddings of Riemannian manifolds into Euclidean space, with the goal of extending it to a broader range of cases and applications in the future. Our framework also extends naturally to the isometric embedding of the Ricci flow, with rigorous error estimates, enabling the visualization of geometric evolutions in intrinsic curvature flows. Numerical experiments support the theoretical analysis by demonstrating the convergence of the method and its effectiveness in simulating isometric embeddings of given Riemannian manifolds as well as Ricci flows.

Yizhou Liang

Finite Element Complexes with Traces Structures: A unified framework for cohomology and bounded interpolation

We consider the cohomology and bounded interpolation of nonstandard finite element complexes, e.g. Stokes, Hessian, Elasticity, divdiv. Compared to the standard finite element exterior calculus, the main challenge is the existence of extra smoothness. We provide a unified framework for finite element complexes with extra smoothness. The trace structure is introduced to derive the bubble complexes in different dimensions (vertices, edges, faces). It is shown that if the bubble complexes in different dimensions are all exact, then the finite element has the correct cohomology. Moreover, the L^2 bounded interpolation can be constructed.

Dongjie Liu

The nonconforming finite element methods for equations with p -Laplace structure

In this talk, we present nonconforming discrete Raviart-Thomas mixed finite element method (dRT-MFEM) for equations with p -Laplace structure: the p -Laplace equation and obstacle problem with p -Laplacian differential operator. Based on the convex control property for the energy density functional, a posteriori and a priori error analysis of dRT MFEM are measured in the new framework. Numerical experiments are presented to validate the efficiency of the proposed method.

Peipei Lu

Two-level hybrid Schwarz methods for Helmholtz equation with high wavenumber

In this talk, we discuss and analyze two-level hybrid Schwarz preconditioners for solving the Helmholtz equation with high wave number in two and three dimensions. Both preconditioners are defined over a set of overlapping subdomains, with each preconditioner formed by a global coarse solver and one local solver on each subdomain. The global coarse solver is based on the localized orthogonal decomposition (LOD) technique, which was proposed originally for the discretization schemes for elliptic multiscale problems with heterogeneous and highly oscillating coefficients and Helmholtz problems with high wave number to eliminate the pollution effect. The local subproblems are Helmholtz problems in subdomains with homogeneous boundary conditions (the first preconditioner) or impedance boundary conditions (the second preconditioner). Both preconditioners are shown to be optimal under some reasonable conditions, that is, a uniform upper bound of the preconditioned operator norm and a uniform lower bound of the field of values are established in terms of all the key parameters, such as the fine mesh size, the coarse mesh size, the subdomain size and the wave numbers. It is the first time to show that the LOD solver can be a very effective coarse solver when it is used appropriately in the Schwarz method with multiple overlapping subdomains. Numerical experiments are presented to confirm the optimality and efficiency of the two proposed preconditioners.

Chupeng Ma

A unified framework for low-rank approximations to multiscale PDEs with optimal error bounds

It is known that any harmonic function in D^* can be approximated in a compact subset D (in the energy norm) by a n -dimensional space with error bound $\mathcal{O}(e^{-cn^\alpha})$. In this talk, I will present an abstract setting for generalizing this property and establish the exponential error bound under two structural conditions – a Caccioppoli type inequality and a weak approximation estimate. This framework applies to a broad class of elliptic PDEs with L^∞ -coefficients in both continuous and discrete, finite element settings, including elasticity, Helmholtz, and Maxwell equations. In particular, for such problems, we obtain the optimal error bound $\mathcal{O}(e^{-cn^{1/(d-1)}})$, with d denoting the spatial dimension of the domain. Applications of this result to the multiscale spectral generalized FEM and low-rank approximations to Green's functions of elliptic PDEs will be presented.

Rui Ma

Adaptive $H(\text{divdiv})$ mixed finite element methods for the Kirchhoff-Love plate bending problem

This talk presents a reliable and efficient residual-based a posteriori error analysis for the symmetric $H(\text{divdiv})$ mixed finite element method for the Kirchhoff-Love plate bending problem with mixed boundary conditions. The key ingredient lies in the construction of boundary-condition-preserving complexes at both continuous and discrete levels. Numerical examples confirm the effectiveness of the a posteriori error estimator and demonstrate the optimal convergence rate under adaptive refinements.

Ira Neitzel

On some aspects of four-dimensional variational data assimilation problems in low-regularity spaces

The talk is concerned with a class of four-dimensional variational data assimilation (4D-VAR) problems for parabolic partial differential equations. Main emphasis will be on nonconvex problems with semilinear PDE constraint with spatial dimension $d = 2$, where we will discuss second order sufficient optimality conditions. Particular emphasis will be placed on the discussion of regularity of solutions to the underlying PDE with certain L^p initial data, since pointwise observations of the state in the cost functional require continuity of the state with respect to the spatial variable.

This is joint work with P. Castro and J. C. de los Reyes.

Daniel Peterseim

Metric-driven numerical methods for nonlinear eigenvalue problems

We introduce the concept of metric-driven numerical methods for the numerical solution of constrained minimization problems and the associated Euler-Lagrange equations, including nonlinear PDE eigenvalue problems. The central idea is to formulate iterative schemes by defining

gradients with respect to problem-adapted metrics, so that the choice of metric directly determines the numerical method and its convergence behavior. Interestingly, the choice of metric also induces approximation spaces with enhanced properties, in particular for problems with limited regularity or heterogeneous coefficients. From this perspective, we recover the Localized Orthogonal Decomposition (LOD) as a well-known class of approximation spaces derived from a new viewpoint. We apply the approach to the computation of ground states of spin-orbit coupled Bose–Einstein condensates.

This is joint work with P. Henning and L. Huynh (Bochum).

Thomas Richter

Hybrid Neural Network / Finite Element Approaches for Solving Partial Differential Equations

Classical numerical methods for solving partial differential equations (PDEs) are highly mature and efficient, yet they reach their limits when confronted with the complexity of real-world applications. Prominent examples include aerodynamics, aeroelasticity, and climate modeling, where resolving all relevant spatial and temporal scales remains far beyond current computational capabilities.

Over the past decades, a wide range of neural network-based approaches has been proposed as alternatives or complements to traditional solvers. While such methods have achieved remarkable success in data-rich settings as in weather forecasting, they have so far fallen short of their promises in engineering applications, where data are limited and reliability, stability, and interpretability are essential.

Hybrid neural network / finite element approaches offer a structurally different perspective. They can be interpreted as multiscale methods that enrich classical simulations with fine-scale information that cannot be resolved by the underlying finite element discretization alone. This viewpoint opens the door to rigorous theoretical analysis and enables, at least to some extent, guarantees on stability and accuracy.

In this talk, we first demonstrate the potential of hybrid solvers to significantly increase computational efficiency. We then present recent analytical results that provide insight into their mathematical properties and limitations.

Christian Rieger

Semi-Lagrange methods on manifolds

In this talk we will discuss the use of meshfree and kernel-based methods for solving partial differential equations on manifolds. The main focus will be on first order equations and semi-Lagrange schemes. Though, the method is well-established the error analysis has been based on some restrictive stability assumptions. I will present a new perspective on these stability assumptions via local polynomial reproductions on (parts of) smooth algebraic varieties. These results have multiple other applications beyond the setting discussed in this talk. Finally, numerical evidence supporting the theoretical error analysis will be provided.

This talk is based on joint works with: Thomas Hangelbroek (University of Hawai'i) and Grady Wright (Boise State University)

Oliver Sander

Numerical Analysis of Geometric FE Discretizations of Cosserat Materials

Cosserat materials are nonstandard models of continuum mechanics where every material point is equipped with a rotation in addition to its position. These rotations allow to model, e.g., shear motion of shells and lattice deformations of crystals, and they introduce an internal length scale in the model. The numerical simulation of Cosserat materials is challenging because the rotations live in the nonlinear space $SO(3)$. We introduce geometric finite elements, a generalization of piecewise polynomial functions that allows to approximate $SO(3)$ -valued functions in a conforming way. For these nonlinear finite element methods we show existence theory, discretization error bounds, and a few numerical results.

Robert Scheichl

Localised Model Reduction via Generalised Finite Elements

Multiscale Spectral Generalized Finite Element Methods (MS-GFEM) are a powerful new discretisation method for general variational problems that satisfy a Gårding-type inequality, including strongly non-Hermitian problems. The construction of optimal approximation spaces is localised and requires no a priori regularity assumptions. The global approximation error is controlled by the local errors, which are rigorously shown to decay nearly exponentially. The optimality hinges on an SVD of the local restriction operator in a suitable, coefficient-dependent inner product on an oversampled patch. Compactness of this operator in the space of a -harmonic functions guarantees spectral accuracy akin to Weyl asymptotics for the Laplacian. As such, MS-GFEM can be seen as an “hp-version” of Localized Orthogonal Decomposition (LOD). Given the coefficient function the local approximation spaces can be constructed efficiently. In this talk, I will show how we can use Grassmannian interpolation of the resulting local approximation spaces on sparse grids in high dimensions to derive localized model order reduction methods that inherit the nearly exponential spatial and parametric convergence of MS-GFEM and of sparse grids, respectively. I will show some numerical experiments confirming the theoretical results in the context of elliptic problems.

Johannes Storn

The singular Zienkiewicz tetrahedron

We extend the 2D singular Zienkiewicz element to three dimensions. This leads an H^2 conforming finite element in 3D that uses rational polynomials as basis functions. These basis functions can be integrated exactly via an iterative scheme.

Chunmei Su

Temporal high-order parametric finite element methods for geometric flows

The quality of the mesh is crucial for simulating curvature flows, as standard approaches may fail due to mesh distortion. We present a series of temporal high-order parametric finite element methods based on the BGN formulation for solving various types of flows involving curves and surfaces. Extensive numerical experiments demonstrate the anticipated high-order accuracy while maintaining favorable mesh quality throughout the evolution process.

Hongpeng Sun

A stochastic preconditioned Douglas-Rachford splitting method for saddle-point problems

We propose and study a stochastic and relaxed preconditioned Douglas- Rachford splitting method to solve saddle-point problems that have separable dual variables. We prove the almost sure convergence of the iteration sequences in Hilbert spaces for a class of convex-concave and nonsmooth saddle-point problems. We also provide the sublinear convergence rate for the ergodic sequence concerning the expectation of the restricted primal-dual gap functions. Numerical experiments show the high efficiency of the proposed stochastic and relaxed preconditioned Douglas- Rachford splitting methods. This is a joint work with Yakun Dong and Kristian Bredies.

Ngoc Tien Tran

Finite element discretizations for fully nonlinear Hamilton-Jacobi-Bellman equations

In this talk, we present recent developments in the design of finite element discretizations for fully nonlinear Hamilton-Jacobi-Bellman equations with a minimal residual strategy. The residual is based on the Alexandrov-Bakelman-Pucci maximum principle for strong solutions and provides guaranteed error control in the maximum norm. Adaptive mesh-refining strategies are discussed. As applications, we numerically approximate singular solutions to the Pucci and Monge–Ampère equations.

Axel Voigt

Arrested coarsening in two-phase systems and characterization of resulting morphologies

We consider two-phase systems modelled by Cahn-Hilliard-Navier-Stokes equations and add various active components at the interface or within the bulk phases, which lead to attested coarsening. We analyse the resulting morphologies using various geometric and topological measures and use them to compare the different hydrodynamic active field theories. As one such measure we explore persistence homology.

Thomas Wick

Parallel Monolithic Matrix-Free Solvers for Coupled Problems

In this presentation, we develop an adaptive finite element framework for quasi-static, fluid-filled phase-field fractures that combines semi-smooth Newton methods, fixed-stress iterative coupling, and parallel matrix-free geometric multigrid preconditioning. The geomechanics subproblem, with displacement and phase field as primary variables, is solved by a semi-smooth combined Newton method based on a primal–dual active set formulation. The resulting linear systems are treated with the generalized minimal residual method and matrix-free geometric multigrid preconditioners on locally refined meshes, with a local smoothing approach and careful treatment in the active set updates. The pressure equation is coupled to the geomechanics system via a fixed-stress iterative scheme which iterates until both subproblem residuals

are sufficiently minimized, and each flow subproblem is likewise preconditioned by matrix-free geometric multigrid on the same adaptive meshes. The overall computational framework is demonstrated on a set of two- and three-dimensional benchmark problems, highlighting its robustness, efficiency, and accuracy for simulating fluid-filled phase-field fractures with local mesh refinement. Finally, ongoing and related work will be briefly discussed as well.

Xiaoping Xie

A Reynolds-semi-robust globally divergence-free E-HDG method for variational data assimilation problems of Navier-Stokes Equations

We develop a globally divergence-free embedded hybrid discontinuous Galerkin (E-HDG) method, combined with an implicit-explicit scalar auxiliary variable (IMEX-SAV) scheme, within a variational framework for data assimilation of the unsteady incompressible Navier-Stokes equations. For spatial discretization, the E-HDG scheme uses piecewise polynomials of degrees k ($k \geq 1$) and $k - 1$ respectively for the approximations of the velocity and the pressure in the interior of elements, and uses continuous polynomials of degree k for the velocity trace and piecewise polynomials of degree k for the pressure trace on the interfaces of elements. This construction yields globally divergence-free approximations of both the velocity and the initial value. For temporal discretization, the IMEX-SAV scheme is adopted to ensure the unconditional energy stability of the velocity field. Under suitable assumptions, the method achieves a Reynolds-semi-robust convergence rate of order k for the velocity error in the L^2 -norm. Finally, numerical experiments are conducted to verify the theoretical results.

Xuejun Xu

Interior Eigensolvers with Domain Decomposition

In this talk, we shall propose an efficient and parallel interior eigensolver for computing eigenvalues of symmetric elliptic operators, which may compute efficiently more interior multiple and clustered eigenvalues other than only the first several smallest eigenvalues. The new method is parallel in two ways: one is to solve the correction equations inexactly by the two-level additive overlapping Schwarz preconditioner, the other is to solve different clusters of eigenvalues simultaneously in different computers. A rigorous theoretical analysis is given for the new method, which reveals that the convergence factor is independent of the fine mesh size and the internal gaps among the target interior clustered eigenvalues, depending on the distance from both ends of the target eigenvalues to others. Meanwhile, more subdomains lead to the better convergence factor. Numerical results supporting our theory shall be given. This is a joint work with Qigang Liang.

Ye Zhang

Neural network yields regularization for ill-posed inverse problems

In this talk, we establish universal approximation theorems for neural networks applied to general nonlinear ill-posed operator equations. In addition to the approximation error, the measurement error is also taken into account in our error estimation. We introduce the expanding neural network method as a novel iterative regularization scheme and prove its regularization

properties under different a priori assumptions about the exact solutions. Within this framework, the number of neurons serves as both the regularization parameter and iteration number. We demonstrate that for data with high noise levels, a small network architecture is sufficient to obtain a stable solution, whereas a larger architecture may compromise stability due to overfitting. Furthermore, under standard assumptions in regularization theory, we derive convergence rate results for neural networks in the context of variational regularization. Several numerical examples are presented to illustrate the robustness of the proposed neural network-based algorithms.

Barbara Zwicknagl

Variational models for pattern formation in materials: From frustrated spin systems to shape-memory alloys

We discuss recent results on variational models for pattern formation in certain materials, focussing on helimagnetic compounds and shape-memory alloys. Precisely, we consider functionals that contain a non-convex multi-well bulk energy favouring finitely many gradients, and a singular perturbation in terms of the total variation of the second derivative. Relations to frustrated spin models from statistical mechanics are studied, and the scaling laws for the infima of the energies in the case of incompatible boundary conditions are derived. Extensions to the vectorial case and the occurrence of topological singularities are briefly discussed.

This talk is based on joint works with J. Ginster, M. Koser, A. Rland, and A. Tribuzio.